

Improved positivity bound for Deep Inelastic Scattering on transversely polarized nucleon

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Abstract

The positivity bound for the transverse asymmetry A_2 may be improved by making use of the fact, that the state of a photon and a nucleon with total spin 3/2, does not participate to the interference. The bound is therefore useful in the case of a longitudinal asymmetry small (say, at low x) or negative (like in the neutron case).

Positivity is playing a very important role in constraining various spin-dependent observables, in particular by providing a bound for the transverse asymmetry in polarized Deep Inelastic Scattering (DIS). It is a well-known condition established long time ago and based on an extensive study by Doncel and de Rafael [1], written in the form

$$|A_2| \leq \sqrt{R} , \quad (1)$$

where A_2 is the usual transverse asymmetry and $R = \sigma_L/\sigma_T$ is the standard ratio in DIS of the cross section of longitudinally to transversely polarized off-shell photons. It reflects a non-trivial positivity condition one has on the photon-nucleon helicity amplitudes. By substituting photons for gluons, we found earlier[2], that the similar bound holds for the various matrix elements for longitudinal gluons in a nucleon [3]

$$|\Delta G_T(x)| \leq \sqrt{1/2G(x)G_L(x)} . \quad (2)$$

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However, this bound can be rederived in line with the positivity bound in the quark case, known as Soffer inequality [4],

$$|h_1(x)| \leq q_+(x) = \frac{1}{2}[q(x) + \Delta q(x)] , \quad (3)$$

by making the substitution in Eq.(2), $G(x) \rightarrow G_+(x) = \frac{1}{2}[G(x) + \Delta G(x)]$, and providing a stronger restriction, especially when the gluon helicity distribution $\Delta G(x)$ is small or even negative. Coming back to the photon case, if A_1 denotes the asymmetry with longitudinally polarized nucleon, we are led to

$$|A_2| \leq \sqrt{R(1 + A_1)/2} , \quad (4)$$

a stronger bound than Eq.(1). In the present paper we will show that this is really the case, using a transparent physical approach, and we will comment on, why, we think the weaker bound was used up to now.

We start with the following expressions for the various photon-nucleon cross-sections in terms of the matrix elements describing the transition from the state $|H, h\rangle$ of a nucleon with helicity h and a photon with helicity H , to the unobserved state $|X\rangle$

$$\begin{aligned} \sigma_T^\pm &= \sum_X | \langle +1/2, +1 | X \rangle |^2 \pm | \langle +1/2, -1 | X \rangle |^2 , \\ \sigma_L &= \sum_X | \langle +1/2, 0 | X \rangle |^2 = \sum_X | \langle -1/2, 0 | X \rangle |^2 , \\ \sigma_{LT} &= 2Re \sum_X \langle +1/2, +1 | X \rangle \langle -1/2, 0 | X \rangle . \end{aligned} \quad (5)$$

Note that while longitudinal and transverse cross-sections are symmetric with respect to the reverse of the nucleon and photon helicities, this is not the case for the interference term. The reason is very simple: the opposite helicities of photon and nucleon correspond to their spins parallel, so that the angular momentum of the state $|X\rangle$ has its maximum value $3/2$. The amplitude, which could possibly interfere with it to produce the transverse asymmetry, should have the same total angular momentum of the state $|X\rangle$. This is however impossible, as the flip of the one of the helicities would require another one to exceed its maximal possible value, in order to keep the angular momentum of $|X\rangle$ the same. Therefore the interference, responsible for A_2 ,

does not occur. This is quite a general reason, for the occurrence of the + helicity configurations in all the cases considered above.

We are now ready to write down the Cauchy-Schwarz inequality as

$$\sum_X | \langle +1/2, +1 | X \rangle \pm a \langle -1/2, 0 | X \rangle |^2 \geq 0 , \quad (6)$$

where a is a positive real number. By making use of the definitions (5) and after the standard minimization with respect to the choice of a , one immediately arrives at

$$|\sigma_{LT}| \leq \sqrt{\sigma_L \sigma_T^+} , \quad (7)$$

leading directly to (4). The use of the new bound is resolving partially the puzzle, why the measured A_2 is such a small quantity. The fact, that the bound (1) is far from being saturated is obvious at low x in the proton case, because, according to (4), it should be decreased by a factor $\sqrt{2}$ due to the small longitudinal asymmetry. The bound under consideration is even more useful with a negative longitudinal asymmetry, like in the neutron case. Recall that for a pure 3/2 configuration we have $A_1 = -1$ which implies $A_2 = 0$

One should note finally, that this result is actually coming from the original papers [5, 1], while it was somehow weakened and transformed to a more suitable form Eq.(1), because one was willing to exclude A_1 which was poorly known twenty years ago. To be convince of that, one should look at Eq.(2.40a) in [1], which was, in fact, already contained in [5].

To conclude, we rederived a known, but so far forgotten stronger bound for the transverse asymmetry in polarized DIS.

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